Spring School in Nonlinear Partial Differential Equations
Brussels, May 30–June 6, 2012

Organizers: D. Bonheure (ULB), A. Ponce, J. Van Schaftingen (UCL), and C. Troestler (UMONS).

Scientific committee: H. Berestycki, J.P. Gossez, E. Lami Dozo, J. Mawhin, C. Stuart, M. Willem.

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Contents

1 Program

Wed 30	Thu 31	Fri 1	Sat 2
8:30–9:00 *Registration*	8:30–9:30 Christ – L2	8:30–10:30 Ball – L2	9:00–9:45 Stuart
9:10–10:40 Ghoussoub – L1	9:30–10:30 Ghoussoub – L2		10:00–10:45 Willem
Coffee break	*Coffee break*	*Coffee break*	*Coffee break*
11:00–12:30 Tanaka – L1	11:00–12:30 Tanaka – L2	11:00–12:30 Ghoussoub – L3	11:15–12:00 Gloria
			12:15–13:00 Squassina
Lunch	*Lunch*	*Lunch*	
14:00–15:30 Christ – L1	14:00–16:00 Ball – L1	14:00–15:30 Christ – L3	
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Rooms:

- Lectures and conferences: FORUM G
- Contributed talks: FORUM G, E and H

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Lunch	*Lunch*	*Lunch*
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16:00–18:00 CT – session 4	16:30–17:15 Serra	
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Rooms:

- Lectures and conferences: Solvay amphitheater
- Contributed talks: Solvay amphitheater and rooms NO707, NO708

Click on a name to go to the abstract.

2 Abstracts

2.1 Lectures

The Mathematics of Liquid Crystals

John Ball

Liquid crystals represent a vast and diverse class of materials which are intermediate between isotropic liquids and crystalline solids. The lecture will describe these fascinating materials and what mathematics, in particular the calculus of variations and partial differential equations, can say about them. The Landau-de Gennes theory, in which the orientational order of the molecules is represented by a matrix-valued order parameter, will be emphasised, together with its relation to the simpler Oseen-Frank theory.

Arithmetic progressions and near equality in affine invariant inequalities

Michael Christ

These lectures will survey progress in understanding those functions which extremize, and especially those which nearly extremize, certain inequalities. Among these are Young's convolution inequality, the Riesz-Sobolev rearrangement inequality, the Brunn-Minkowski inequality, and an inequality for the Radon transform. The group of all invertible affine transformations of Euclidean space is a symmetry group of each of these inequalities. Arithmetic progressions and the geometry of Euclidean space are at the heart of the analysis.

Self-dual variational calculus: From existence and uniqueness to homogenization and control

Nassif Ghoussoub

We describe how the theories of self-dual Lagrangians and anti-symmetric Hamiltonians can be applied to give variational proofs for the existence and uniqueness of solutions to various PDEs and evolution equations. It is also applied to the study of inverse problems, optimal control, and the homogenization of equations involving monotone vector fields. It also leads to a self-dual version of Brenier's polar decomposition for general vector fields.

TBA

Peter A. Markowich

Equivariant Bifurcation in Geometric Variational Problems

Paolo Piccione

The first lecture will be about some abstract results on the equivariant implicit function theorem and equivariant bifurcation. In the second lecture I will discuss some applications to the case of minimal and CMC embeddings in Riemannian geometry. The third lecture will be about applications to the Yamabe problem and its generalizations.

Topics in geometric spectral theory [1]

Iosif Polterovich

Geometric spectral theory is a field of mathematics at the interface of partial differential equations, geometry and functional analysis. It has a long and fascinating history, going back to the experiments of Chladni with vibrating plates, to the groundbreaking work of Rayleigh on the theory of sound and to Kac's celebrated question "Can one hear the shape of a drum?". The aim of the mini-course is to present basic notions and fundamental results of geometric spectral theory, as well as to discuss some recent developments and open problems.

Stability of solitons and multi-solitons via the variational method. Application to the Gross-Pitaevskii equation

Didier Smets

Stability of solitary waves has long been an intensive area of research. Modern methods include a) inverse scattering transforms and/or Riemann-Hilbert problems, when the problem at hand is integrable, and b) the more robust variationnal method, in other cases. In these lectures, I will start by reviewing some presumably interesting questions related to solitary waves that arise naturally in a number of Hamiltonian equations, like the (generalized) Korteweg - de Vries (gKdV) equation, the Non Linear Schrodinger equations (NLS), and the Toda lattice equations. Even though integrable methods can be quite powerful (e.g. notably for the integrable KdV), in some cases, even for integrable equations, the variationnal method has revealed itself as the only one yet to provide tractable qualitative information. I will next focus on the Gross-Pitaevskii (GP) equation in 1D, a defocusing NLS equation with a non vanishing condition at infinity. Equation (GP) is known to be integrable since the work of Zakharov and Shabat in 1973. Yet the variationnal method seems the method of choice. I will mainly discuss three

[1] The minicourse of Prof. Polterovich will take place from June 25 to 29, 2012.

issues for (GP) : 1) orbital stability of solitary waves: on this occasion I will refer to the powerful and well-known methods of Benjamin/Cazenave-Lions/Grillakis-Shatah-Strauss. 2) orbital stability of multi-solitons: this concerns the "orbital" stability of a train of well-separated solitons. Stability does not necessarily holds in that situation, and the orbital stability (purely variationnal) of single solitons needs to be combined with dynamical information (here a monotonicity formula). 3) asymptotic stability of solitary waves: this is a much more subtle (and stronger) form of stability which requires a good understanding of the linearized flow around a solitary wave.

Variational approach to nonlinear elliptic problems

Kazunaga Tanaka

We consider the existence of least energy solutions of nonlinear elliptic problems via variational approach. We deal with both scalar problems and systems of elliptic equations. We also give applications to singularly perturbed Schrodinger equations. More precisely, in first half of my lecture, we will mainly deal with scalar field equations and give another proof of the fundamental result of Berestycki and Lions using Mountain Pass theorem (without introducing constraint problems). We will also introduce some technique from the theory of dynamical systems to deal with $N = 1$ (especially for systems). If time allows, we will also deal with singular perturbation problems.

2.2 Invited talks

Strong ellipticity of nonlinear elastic materials and discrete homogenization

Antoine Gloria

This talk is concerned with the strong ellipticity of nonlinear elastic materials obtained by the periodic or stochastic homogenization of discrete systems. Since the seminal analysis by Geymonat, Müller and Triantafyllidis [2], it is known that strict strong ellipticity can be lost by periodic homogenization in nonlinear elasticity. I'll show how to extend their analysis to the case of discrete homogenization on the lattice $\mathbb{Z}^d$, both with periodic and stochastic interactions. To this aim I'll introduce a stochastic discrete variant of the Bloch transform, relying on stationarity rather than on periodicity. To illustrate the main result, I'll show in some periodic case that a microscopic bifurcation can occur and implies a loss of strong ellipticity for the homogenized material. Yet, for another type of interactions relevant to polymer materials, a simple argument implies that the homogenized material is strictly strongly elliptic in the periodic case. In the case when the lattice $\mathbb{Z}^d$ is replaced by a stochastic lattice — as it is suitable for some discrete models for rubber studied in [1, 3] —, the strict strong ellipticity also holds under a perturbation assumption. Numerical simulations support the validity of these assumptions.

References

[1] R. Alicandro, M. Cicalese, and A. Gloria. Integral representation results for energies defined on stochastic lattices and application to nonlinear elasticity. *Arch. Ration. Mech. Anal.*, 200(3):881–943, 2011.

[2] G. Geymonat, S. Müller, and N. Triantafyllidis. Homogenization of nonlinearly elastic materials, microscopic bifurcation and macroscopic loss of rank-one convexity. *Arch. Rat. Mech. Anal.*, 122:231–290, 1993.

[3] A. Gloria, P. Le Tallec, and M. Vidrascu. Comparison of network-based models for rubber. 2012. Preprint, http://hal.archives-ouvertes.fr/hal-00673406.

Radial solutions of Neumann problems for periodic perturbations of the extrinsic mean curvature operator on an annulus

Jean Mawhin

Using a Lusternik-Schnirelman type multiplicity result for some indefinite functionals due to Szulkin, the existence of at least $n+1$ geometrically distinct radial solutions is proved for the homogeneous Neumann problem on an annulus $A(\rho, R)$ in $\mathbb{R}^N$ centered at 0 associated to systems of the form

$$\nabla \cdot \left(\frac{\nabla w_i}{\sqrt{1 - \sum_{j=1}^{n} \|\nabla w_j\|^2}} \right) + \partial_{w_i} G(\|x\|, w) = h_i(\|x\|), \qquad (i = 1, \dots, n),$$

involving the extrinsic mean curvature operator in a Minkovski space, a potential G periodic in each component of w and a function h such that

$$\int_{\rho}^{R} h(r) r^{N-1} \, \mathrm{d}r = 0.$$

On a conjecture of De Giorgi concerning nonlinear wave equations

Enrico Serra

We prove a conjecture by De Giorgi, which states that global weak solutions to the Cauchy problem associated to certain non-linear wave equations can be obtained as limits of minimizers of suitable functionals of the calculus of variations. There is no restriction on the nonlinearity exponent, and the method is easily extended to more general equations.

Some results on the quasi-linear Schrodinger equation

Marco Squassina

We discuss some recent results about the ground states and the orbital stability-instability for the quasi-linear Schrodinger equation. We also present some results about the numerical computation of ground states and about a bifurcation phenomenon occurring for a related constrained minimization problem. The results were obtained in joint papers with (in alphabetical order) Marco Caliari, Mathieu Colin, Francesca Gladiali, Christopher Grumiau, Louis Jeanjean and Christophe Troestler.

Asymptotic Linearity and Hadamard Differentiability

Charles A. Stuart

Motivated by the study of solutions of second order nonlinear elliptic equations in the usual Sobolev spaces $W^{2,p}(\mathbb{R}^N)$ for $1 \leqslant p < \infty$, we present a variant of the standard notion of asymptotic linearity of a mapping $M : X \to Y$ acting between Banach spaces X and Y. For the associated inversion, $M^*(u) = u^2 M(u/u^2)$, this new property is equivalent to Hadamard differentiability at 0. New results about bifurcation for Hadamard differentiable problems then lead to conclusions about asymptotic bifurcation for nonlinear elliptic equations on $\mathbb{R}^N$. They are quite different from what occurs for the same equation on a bounded domain with Dirichlet boundary condition.

Symmetry of ground states of coupled Schrödinger equations

Michel Willem

We consider partial symmetry of the least energy solutions to some nonlinear Schrödinger systems. In the attractive case we show that all components of the least energy solutions possess the foliated Schwarz symmetry with respect to the same point.

2.3 Contributed talks

Generalized Schrödinger-Poisson type systems

Pietro d'Avenia

We consider generalized Schrödinger-Poisson systems in a smooth and bounded domain with Dirichlet boundary conditions. We discuss existence and nonexistence results which have been obtained in collaboration with A. Azzollini and V. Luisi.

Existence and Multiplicity Results for some Elliptic Systems in Unbounded Cylinders

Sara Barile

The aim of the communication is to present some recent results, obtained in [1], on the existence and multiplicity of solutions for the following nonlinear elliptic system of Lane-Emden type on unbounded cylinders

$$\begin{cases} -\Delta u = \operatorname{sgn}(v)|v|^{p-1} & \text{in } \Omega, \\ -\Delta v = -\lambda \operatorname{sgn}(u)|u|^{1/(p-1)} + f(x,u) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega = \widetilde{\Omega} \times \mathbb{R}^{N-m} \subseteq \mathbb{R}^N$, $N-m \geqslant 2$, $\widetilde{\Omega} \subseteq \mathbb{R}^m$ $(m \geqslant 1)$ is open and bounded, $\lambda \in \mathbb{R}$, $1 < p \leqslant 2$ and $f : \Omega \times \mathbb{R} \to \mathbb{R}$ satisfies suitable growth's assumptions at the origin and at infinity. The search of weak solutions for the above system can be reduced to the study of critical points of the energy functional I associated to the equivalent fourth order quasilinear elliptic problem

$$\begin{cases} -\Delta(-\Delta u)^{1/(p-1)} + \lambda u^{1/(p-1)} = f(x,u) & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega. \end{cases}$$

In order to overcome the lack of compactness of the problem, by means of a result by P.L. Lions [3], we prove some compact imbeddings of suitable "partially" spherically symmetric Sobolev spaces. So, under spherically symmetric assumptions on f in $\mathbb{R}^{N-m}$, by the Principle of Symmetric Criticality by Palais, we can look for critical points of I constrained on such spaces. In particular, for $\lambda < 0$ and suitable p's, by following an approach introduced in [2], we consider a finite-dimensional decomposition of such spaces. A direct application of Mountain Pass Theorem allows us to establish the existence of one nontrivial weak solution of the system. Under further hypotheses of symmetry of f, we prove a multiplicity result by the Symmetric Version of Mountain Pass Theorem.

References

[1] S. Barile and A. Salvatore, *Existence and Multiplicity Results for some Elliptic Systems in Unbounded Cylinders*, Preprint.

[2] A.M. Candela and G. Palmieri, *Infinitely many solutions of some nonlinear variational equations*, Calc. Var. 34 (2009), 495–530.

[3] P.L. Lions, *Symétrie et compacité dans les espaces de Sobolev*, J. Funct. Anal. 49 (1982), 315–334.

On the solutions set to some Robin problems

Elvise Berchio

In a smooth bounded domain $\Omega \subseteq \mathbb{R}^n$ ($n \geqslant 2$), consider the problem

$$\begin{cases} -\Delta u = \lambda g(u) & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ u_\nu + cu = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where u_ν denotes the outer normal derivative of u on $\partial\Omega$, $c, \lambda > 0$ and g is a suitable positive, increasing and convex function.

We discuss the extension to (1) of some well-known results for semilinear elliptic Dirichlet problems. A particular attention is devoted to the regularity of the so-called extremal solution and, for critical growth power type nonlinearities, to the existence of a second mountain-pass solution. The extension discovers some new phenomena and show how small values of the boundary parameter play against strong growth nonlinear terms and large space dimensions.

The results presented are contained in the papers [1] and [2].

References

[1] E. Berchio, On the second solution to a critical growth Robin problem, J. Math. Anal. Appl., vol. 389 (2012), 950-967.

[2] E. Berchio, F. Gazzola, D. Pierotti, Gelfand type elliptic problems under Steklov boundary conditions, Ann. Inst. Henri Poincaré, Analyse non Linéaire, vol. 27 (2010), 315-33.

Existence and multiplicity results for Neumann problems with variable exponents [2]

Maria-Magdalena Boureanu

We start by shortly describe the isotropic and anisotropic spaces with variable exponents, together with their main properties. Our study is developed in the framework of these spaces and we are discussing the existence and multiplicity of weak solutions for general elliptic problems with variable exponents and Neumann boundary conditions. More exactly, we are interested in problems involving $p(\cdot)$-Laplace type operators or $\vec{p}(\cdot)$-Laplace type operators, where $p \in C\big(\Omega; (1,\infty)\big)$, $\vec{p} \in C\big(\Omega; (1,\infty)\big)^N$ and $\Omega \subseteq \mathbb{R}^N$ $(N \geqslant 2)$ is a bounded domain with smooth boundary. Our goal is not only to introduce the new results, but also to present and comment the connections that appear between them.

Local behaviour of singular solutions for nonlinear elliptic equations in divergence form

Barbara Brandolini

We consider the following class of nonlinear elliptic equations

$$-\mathrm{div}(\mathcal{A}(|x|)\nabla u) + u^q = 0 \quad \text{in } B_1(0) \setminus \{0\},$$

where $q > 1$ and $\mathcal{A}$ is a positive $C^1(0,1]$ function which is regularly varying at zero with index ϑ in $(2-N, 2)$. We prove that

[2] The author was supported by the Romanian Grants CNCSIS–PCCE–08/2010 and CNCS-PNII-ID-PCE-47/2011

all isolated singularities at zero for the positive solutions are removable if and only if $\Phi \notin L^q(B_1(0))$, where Φ denotes the fundamental solution of $-\mathrm{div}(\mathcal{A}(|x|)\nabla u) = \delta_0$ in $\mathcal{D}'(B_1(0))$ and δ_0 is the Dirac mass at 0. Moreover, we give a complete classification of the behaviour near zero of all positive solutions in the more delicate case that $\Phi \in L^q(B_1(0))$. We also establish the existence of positive solutions in all the categories of such a classification. Our results apply in particular to the model case $\mathcal{A}(|x|) = |x|^\vartheta$ with $\vartheta \in (2-N, 2)$.

Positive solutions of the Dirichlet problem for the one-dimensional Minkowski-curvature equation

Isabel Coelho

We discuss existence and multiplicity of positive solutions of the Dirichlet problem for the quasilinear ordinary differential equation

$$-\left(\frac{u'}{\sqrt{1-u'^2}}\right)' = f(t,u),$$

under various configurations of the right-hand side $f = f(t,s)$. Depending on the behaviour of f near $s = 0$, we will prove existence, multiplicity and nonexistence of positive solutions. In general, the positivity of f is not required. All results are obtained by reduction to an equivalent non-singular problem, to which variational or topological methods apply in a rather standard way. This is a joint work with P. Omari, F. Obersnel and C. Corsato.

Positive solutions of the Minkowski-curvature equation

Chiara Corsato

In this talk we first present some results about existence and multiplicity of positive solutions for the mixed boundary-value problem

$$\begin{cases} -\left(\dfrac{r^{N-1}u'}{\sqrt{1-(u')^2}}\right)' = r^{N-1}f(r,u) & \text{in }]0,R[, \\ u'(0)=0, \quad u(R)=0, \end{cases}$$

which are the positive radial solutions of the corresponding N-dimensional Dirichlet problem in a ball. We discuss various configurations of $f = f(r, s)$, including superlinear, or sublinear, or sub-superlinear behaviour near $s = 0$. Here variational techniques are used.

This paves the way for studying the existence of positive solutions of the Dirichlet problem

$$\begin{cases} -\text{div}\left(\nabla u/\sqrt{1-|\nabla u|^2}\right) = f(x, u, \nabla u) \quad \text{in } \Omega, \\ u = 0 \quad \text{on } \partial\Omega, \end{cases}$$

where Ω is a general bounded domain in $\mathbb{R}^N$ and the right-hand side of the equation is allowed to depend on the gradient of the solution. Here the approach makes use of topological degree methods.

References

[1] I. Coelho, C. Corsato, F. Obersnel and P. Omari, Positive solutions of the Dirichlet problem for the one-dimensional Minkowski-curvature equation, preprint (2012).

[2] I. Coelho, C. Corsato and S. Rivetti, Multiple positive solutions of the Dirichlet problem for the Minkowski-curvature equation in a ball, in preparation.

Subcritical and supercritical Klein-Gordon-Maxwell equations without Ambrosetti-Rabinowitz condition [3]

Patrícia L. Cunha

In this work we present some results on the existence of positive and ground state solutions for the nonlinear Klein-Gordon-Maxwell equations. We introduce a general nonlinearity with subcritical and supercritical growth which does not require the usual Ambrosetti-Rabinowitz condition. The proof is based on variational methods and perturbation arguments.

[3] Supported by CAPES/Brazil.

Wrinkling of thin membranes laying on fluid substrates under small compression [4]

Bertrand Desmons

We are interested in the asymptotic profile of clamped thin membranes laying on fluid substrates like water, and slightly compressed. We suppose that the film is inextensible and covers a bounded domain $\Omega \subseteq \mathbb{R}^2$ without compression. We show that, up to a good rescaling, the asymptotic profile of the membrane is given by the first eigenfunction of the fourth-order eigenvalue problem:

$$\Delta^2 u + Ku = \lambda \Delta u \quad \text{on } \Omega;$$
$$u = 0 \text{ on } \partial\Omega, \qquad \partial_\nu u = 0 \text{ on } \partial\Omega,$$

with $K \geqslant 0$ a constant relative to the membrane and the substrate. For $K = 0$ this problem is the well-known problem of plate buckling; we investigate how some results obtained for this problem can be generalized to our eigenvalue problem. Besides, we study the multiplicity of the first eigenvalue for particular domains and values of K. Our results will be illustrated by numerical experiments. This is a joint work with S. Nicaise, Ch. Troestler and J. Venel.

Entire solutions to nonlinear scalar field equations with indefinite linear part

Gilles Evéquoz

We present a joint work with Tobias Weth [1] in which we consider the stationary semilinear Schrödinger equation

$$-\Delta u + a(x)u = f(x, u), \qquad u \in H^1(\mathbb{R}^N),$$

where a and f are continuous functions converging to some limits $a_\infty > 0$ and $f_\infty = f_\infty(u)$ as $|x| \to \infty$. In the indefinite setting where the Schrödinger operator $-\Delta + a$ has negative eigenvalues, we combine a reduction method with a topological argument to prove the existence of a solution of our problem under weak

[4] Supported by a FNRS fellowship grant.

one-sided asymptotic estimates. The minimal energy level need not be attained in this case. In a second part, we prove the existence of a ground-state solution under more restrictive assumptions on a and f. In this case, some of our results also hold when zero lies in the spectrum of the Schrödinger operator.

References

[1] G. Evéquoz and T. Weth, Entire solutions to nonlinear scalar field equations with indefinite linear part, Adv. Nonlinear Stud. **12** (2012) 281–314.

Uniqueness of renormalized solutions to nonlinear parabolic problems with lower order terms

Filomena Feo

In this paper we prove uniqueness results for renormalized solutions to the following class of nonlinear parabolic problems:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \operatorname{div}(a(x,t,u,\nabla u)) \\ \qquad + \operatorname{div}(K(x,t,u)) + H(x,t,\nabla u) = f - \operatorname{div} g & \text{in } Q_T \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases} \tag{1}$$

where Q_T is the cylinder $\Omega \times (0,T)$, Ω is a bounded open subset of $\mathbb{R}^N$, $T > 0$, $p > 1$ and $N \geqslant 2$. Moreover $-\operatorname{div}\big(a(x,t,u,\nabla u)\big)$ is a Leray-Lions operator which is coercive and grows like $|\nabla u|^{p-1}$ with respect to ∇u. The function K and H are Carathéodory functions with suitable assumptions. Finally $f \in L^1(Q_T)$, $g \in \big(L^{p'}(Q_T)\big)^N$ and $u_0 \in L^1(\Omega)$.

The difficulties connected to existence and uniqueness of the solution to this problem are due to the L^1 data and to the presence of the two terms K and H which can induce a lack of coercivity.

For L^1 data and $p > 2 - \frac{1}{N+1}$ in literature is proved the existence of a weak solution to Problem (1) (which belongs to $L^m\big((0,T); W_0^{1,m}(\Omega)\big)$ with $m < \frac{p(N+1)-N}{N+1}$). It is well known that this weak solution is not unique in general (see [7] for a counter-example in the stationary case). We use the framework of renormalized solutions which provides uniqueness and stability properties.

The notion of renormalized solution was introduced in [3, 4] for first order equations and has been adapted for elliptic problems with L^1 data in [5, 6]. This notion was also developed for parabolic equation with L^1 data in [1, 2].

References

[1] D. Blanchard and F. Murat, Renormalized solution for nonlinear parabolic problems with L^1 data, existence and uniqueness. Proc. Roy. Soc. Edinburgh Sect. A, 127:1137–1152, 1997.

[2] D. Blanchard, F. Murat, and H. Redwane, Existence and uniqueness of a renormalized solution for a fairly general class of nonlinear parabolic problems. J. Differential Equations, 177:331–374, 2001.

[3] R.-J DiPerna and P.-L Lions, On the cauchy problem for Boltzmann equations : global existence and weak stability. Ann of Math 130(1):321–366, 1989.

[4] R.-J DiPerna and P.-L Lions, Ordinary differential equations, sobolev spaces and transport theory. Invent. Math, 98:511–547, 1989.

[5] F. Murat, Soluciones renormalizadas de EDP elipticas non lineales. Technical Report R93023, Laboratoire d'Analyse Numérique, Paris VI,1993.

[6] F. Murat, Equations elliptiques non linéaires avec second membre L^1 ou mesure. Compte Rendus du 26e Congrès d'Analyse Numérique les Karellis, 1994.

[7] J. Serrin, Pathological solution of elliptic differential equations. Ann. Scuola Norm. Sup. Pisa Cl. Sci., 18:385–387, 1964.

On the number of solutions of NLS equations with magnetics fields in expanding domains

Giovany Figueiredo

In this paper we look for multiple weak solutions $u : \Omega_\lambda \to \mathbb{C}$ for the complex equation $(-i\nabla - A(\frac{x}{\lambda}))^2 u + u = f(|u|^2)u$ in $\Omega_\lambda = \lambda\Omega$. The set $\Omega \subseteq \mathbb{R}^N$ is a smooth bounded domain, $\lambda > 0$ is a parameter, A is a regular magnetic field and f is a superlinear function with subcritical growth. Our main result relates, for large values of λ, the number of solutions with the topology of the set Ω. In the proof we apply minimax methods and Ljusternick-Schnirelmann theory.

Multiple solutions for nonlinear elliptic equations with fast increasing weight and critical growth

Marcelo Furtado

We are concerned with the existence of rapidly decaying solutions for the equation

$$-\mathrm{div}\big(K(x)\nabla u\big) = f(x,u), \quad x \in \mathbb{R}^N,$$

where $N \geqslant 2$, $K(x) := \exp(|x|^2/4)$ and f has critical growth. In the case $N \geqslant 3$ the obtained solutions are in some sense related to self-similar solutions of a critical heat equation. In the case $N = 2$ the function f is not homogeneous and its behavior at infinity is related to (a properly version of) the Trudinger-Moser inequality. In all the proofs we apply variational methods. The results presented are obtained in some jointly works with J.P.P. Silva & M.S. Xavier and E.S. Medeiros & U.B. Severo.

On the initial-boundary problem associated with a nonlinear dissipative Schrödinger equation

Jesús Montejo Gámez

This contribution is concerned with the three-dimensional local wellposedness of a quantum dissipative Schrödinger equation in bounded domains.

The model, which stems from the Wigner-Fokker-Planck equation, describes interactions of a quantum particle with a termal bath in thermodynamic equilibrium through dissipative and diffusive terms in the wavefunction picture. Most of these terms belong to the so-called "Doebner–Goldin family", which is the largest class of nonlinearities modeling diffusion currents at the Schrödinger level.

The mathematical study of such an equation is carried out by using a nonlinear Gauge transformation which reduces the model to the (purely) logarithmic Schrödinger equation, preserving the observable behaviour. The rigorous treatment of this transformation requires the modulus-argument decomposition of the wavefunction along with the construction (under reasonable assumptions) of solutions to the associated flow-potential hydrodynamic systems.

This is a joint work with P. Guerrero, J. L. López, and J. Nieto.

Lane Emden problems: asymptotic behavior of low energy nodal solutions.

Christopher Grumiau

We study the nodal solutions of the Lane Emden Dirichlet problem

$$\begin{cases} -\Delta u(x) = |u(x)|^{p-1}u(x), & \text{in } \Omega \\ u(x) = 0, & \text{on } \partial\Omega \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^2$ and $p > 1$. We consider solutions u_p satisfying

$$p \int_\Omega |\nabla u_p|^2 \to 16\pi e \quad \text{as} \quad p \to +\infty \tag{1}$$

and we are interested in the shape and the asymptotic behavior as $p \to +\infty$.

First we prove that (1) holds for least energy nodal solutions. Then we obtain some estimates and the asymptotic profile of this kind of solutions. Finally, in some cases, we prove that pu_p can be characterized as the difference of two Green's functions and the nodal line intersects the boundary of Ω, for large p. This is a joint work with M. Grossi and F. Pacella.

Multiplicity of solutions to the Dirichlet problem for an supercritical equation with p-laplacian [5]

Sergey Kolonitskii

Consider a boundary problem

$$\begin{cases} -\Delta_p u = u^{q-1} & \text{in } \Omega_R; \\ u > 0 & \text{in } \Omega_R; \\ u = 0 & \text{on } \partial\Omega_R, \end{cases} \tag{1}$$

where $\Omega_R = \{x \in \mathbb{R}^n | R < |x| < R+1\}$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is a p-laplacian. Let p_n^* be a critical Sobolev embedding exponent, i.e. $\frac{1}{p_n^*} = \left(\frac{1}{p} - \frac{1}{n}\right)_+$.

Multiplicity effect for solution of problem (1) was considered in many articles, starting with [1]. The most complete results are obtained in case $p = 2$. It was proved in [3, 2, 4] that for arbitrary $1 < p < \infty$ and $p < q < p_n^*$ for any natural K there exists $R_0 = R_0(n, p, q, K)$ such that for all $R > R_0$ problem (1) has at least K solutions that are nonequivalent up to rotations.

We prove the multiplicity of solutions to problem (1) for $n \geqslant 4$, $n \neq 5$, $1 < p < \infty$ and $p_n^* \leqslant q < p_{n-1}^*$.

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[5] Work is supported by grant RFBR 11-01-00825.

Existence and instability of standing waves with prescribed norm for a class of Schrödinger-Poisson equations

Tingjian Luo

We consider the following Schrödinger-Poisson-Slater equations

$$i\partial_t u + \Delta u - \left(|x|^{-1} * |u|^2\right)u + |u|^{p-2}u = 0 \quad \text{in} \quad \mathbb{R} \times \mathbb{R}^3,$$

where $\frac{10}{3} < p < 6$, and concentrate on the existence and the strong instability of standing wave solutions with prescribed L^2-norm. Our approach is constrained variational. Namely, the standing waves are found by looking to critical points of the associated energy functional on the constraints given by $\{u \in H^1(\mathbb{R}^3) : \|u\|^2_{L^2(\mathbb{R}^3)} = c\}$. For the values $p \in (\frac{10}{3}, 6)$ considered, the functional on the constraint is unbounded from below and the existence of critical points is obtained by a mountain pass argument.

Weakly coupled nonlinear Schrödinger systems: the saturation effect[6]

Liliane Maia

The existence of solutions for a class of saturable weakly coupled Schrödinger system is presented as in [1]. In most of the cases it is shown that least energy solutions have necessarily one trivial component. In addition, sufficient conditions for the existence of a solution with both positive components are found. Moreover, new results on semiclassical limits of standing waves of equations with a variable saturation function are discussed.

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Solutions to locally compact variational problems with nonlocal operators

Michael Melgaard

We study the nonlocal and nonlinear problem

$$L\varphi + V\varphi - |\varphi|^2 * W\varphi = -\lambda\varphi,$$
$$\|\varphi\|_{L^2(\mathbb{R}^3)} = 1,$$

for a large class of potentials V and W. The operator $L = \sqrt{-\alpha^{-2}\Delta + \alpha^{-4}} - \alpha^{-2}$ (the quasirelativistic Laplacian), with α being Sommerfeld's fine structure constant, is a nonlocal, pseudo differential operator of order one. We prove the existence of multiple solutions for two separate cases: (1) unconstrained problem; (2) constrained problem.

On the pure critical exponent problem for the p-Laplacian

Carlo Mercuri

We will discuss about some recent results in collaboration with F. Pacella, dealing with the following boundary value problem

$$\begin{cases} -\Delta_p u = |u|^{p^*-2}u & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where $1 < p < N$, $p^* := Np/(N-p)$ is the critical Sobolev exponent, $\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian operator defined on

$$\mathcal{D}^{1,p}(\mathbb{R}^N) := \{u \in L^{p^*}(\mathbb{R}^N) : \nabla u \in L^p(\mathbb{R}^N;\mathbb{R}^N)\},$$

Ω is a smooth bounded domain having non-trivial topology, and discrete symmetry.

A minimization problem related to the principal eigenvalue of the s-wave Schrödinger operator

Abbasali Mohammadi

We consider a minimization problem related to the principal eigenvalue of the s-wave Schrödinger operator with an energy-dependent potential. This operator is used to describe collisions of two spineless particles. Our problem is an optimization problem over a rearrangement class of a given function originated from an elliptic eigenvalue problem. Such kinds of optimization problems have many applications in engineering and applied sciences and these problems have been intensively attractive to mathematicians in the past decades. However, it should be mentioned that the majority of the investigated models are linear or nonlinear in their differential operator part. We note that the s-wave Schrödinger is a nonlinear eigenvalue problem which has nonlinear dependence on the eigenparameter and such systems have been under less attention in this field of study. We prove existence of a solution for the minimization problem and uniqueness will be addressed when the domain is a ball by deriving the configuration of the unique solution.

On the solvability of a BVP generated by the Maz'ya–Sobolev inequality[7]

Alexander Nazarov

Denote by $x = (y; z) = (y_1, y'; z)$ a point in $\mathbb{R}^n = \mathbb{R}^m \times \mathbb{R}^{n-m}$, $n \geqslant 3$, $2 \leqslant m \leqslant n-1$. By $\mathcal{P}$ we denote the subspace $\{x \in \mathbb{R}^n : y = 0\}$.

Let Ω be a domain in $\mathbb{R}^n$. By $\mathcal{C}_0^\infty(\Omega)$ we denote the set of smooth functions with compact support in Ω. For $1 \leqslant p < \infty$ we denote by $\dot{W}_p^1(\Omega)$ the closure of $\mathcal{C}_0^\infty(\Omega)$ with respect to the norm $\|\nabla v\|_{p,\Omega}$. Obviously, for bounded domains $\dot{W}_p^1(\Omega) = \overset{o}{W}{}_p^1(\Omega)$.

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By definition, for $0 \leqslant \sigma \leqslant \min\{1, \frac{n}{p}\}$ we put $p^*_\sigma = \frac{np}{n-\sigma p}$. We discuss the attainability of the sharp constant in the so-called *Maz'ya–Sobolev inequality*

$$\big\||y|^{\sigma-1}v\big\|_{p^*_\sigma,\Omega} \leqslant \mathcal{N}(p,\sigma,\Omega)\cdot\|\nabla v\|_{p,\Omega}, \tag{1}$$

which holds true for any $v \in \dot{W}^1_p(\Omega)$ provided

$$\begin{array}{ll} \Omega \text{ is any dom. in } \mathbb{R}^n, & \frac{n(p-m)}{p(n-m)} < \sigma \leqslant 1; \\ \Omega \subseteq \mathbb{R}^n \setminus \mathcal{P}, & p > m,\ \ \sigma \leqslant \min\{\frac{n(p-m)}{p(n-m)}; \frac{n}{p}\},\ \ \sigma \neq 1; \\ \Omega \subseteq \mathbb{R}^n \setminus (\ell \times \mathbb{R}^{n-m}), & p = m,\ \ \sigma = 0 \end{array} \tag{2}$$

(here ℓ is a ray in $\mathbb{R}^m$ beginning at the origin). Note that the case $p < n$, $\sigma = 1$ gives conventional Sobolev inequality.

Remark. Under suitable normalization the extremal function in (1) is a positive solution of the Dirichlet problem to the nonlinear Schrödinger equation

$$-\Delta_p u = \frac{u^{p^*_\sigma - 1}}{|y|^{(1-\sigma)p^*_\sigma}} \quad \text{in} \quad \Omega; \qquad u\big|_{\partial\Omega} = 0.$$

It is easy to see that the sharp constant in (1) for $p < n$ and $0 < \sigma < 1$ does not depend on Ω, and the extremal function *does not exist* for any Ω provided $\Omega \cap \mathcal{P} \neq \varnothing$ and $\dot{W}^1_p(\Omega) \neq \dot{W}^1_p(\mathbb{R}^n)$.

We consider considerably more complicated case $\Omega \cap \mathcal{P} = \varnothing$, $\partial\Omega \cap \mathcal{P} \neq \varnothing$. First, we analyze the attainability of the sharp constant in (1) for Ω being a wedge $\mathcal{K} = K \times \mathbb{R}^{n-m}$ (here K is an open cone in $\mathbb{R}^m$) or a "perturbed" wedge. Here we consider all $1 < p < \infty$ and $0 \leqslant \sigma < \min\{1, \frac{n}{p}\}$. Naturally, we suppose that Ω satisfies (2).

In the second part of the talk we prove the attainability of the sharp constant in (1) in a bounded domain for $p = 2$ and $0 < \sigma < 1$. Note that our requirements on $\partial\Omega$ are considerably weakened comparing with the recent paper of Ghoussoub and Robert.

Remark. For $m = 1$ our problem of interest degenerates in a sence. On the another hand, the problem for $m = n$, corresponding to the Hardy–Sobolev inequality, was investigated in a number of papers. The history of the problem and extensive bibliography was given in the survey [2].

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Multiplicity of solutions of a p-Laplacian equation via the sub-supersolutions method

Benedetta Noris

We consider a class of quasilinear elliptic equations, with a nonlinear term of superlinear and subcritical type. We develop abstract results to prove existence and multiplicity of solutions in presence of sub-supersolutions. As an application, we provide a sign-changing solutions under suitable assumptions. We point out that the standard methods which ensure the existence of a sign-changing solution for semilinear elliptic equations do not extend directly to quasilinear ones. Some of the main obstructions are the lack of an Hilbert space and the lack of regularity, which prevents from applying Morse type arguments. This is a joint work with M.M. Boureanu and S. Terracini.

L^1 estimates for complex elliptic systems of vector fields [8]

Tiago Picon

In this lecture we will discuss a local version of Gagliardo-Nirenberg inequality for complex elliptic systems of vector fields with smooth coefficients. We will present, within the local setup, some L^1 estimates analogous to those known for the de Rham complex on R^N. This is joint work with Jorge Hounie (UFSCar).

[8] Supported by São Paulo Research Foundation — FAPESP.

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Multiplicity of solutions for a biharmonic equation with subcritical or critical growth

Marcos Tadeu de Oliveira Pimenta

We consider the fourth-order problem

$$\begin{cases} \varepsilon^4\Delta^2 u + V(x)u = f(u) + \gamma|u|^{2_{**}-2} & \text{in } \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases}$$

where $\varepsilon > 0$, $N \geqslant 5$, V is a positive continuous potential, f is a function with subcritical growth and $\gamma \in \{0, 1\}$. We relate the number of solutions with the topology of the set where V attain it minimum values. We consider the subcritical case $\gamma = 0$ and the critical case $\gamma = 1$. In the proofs we apply Ljusternik-Schnirelmann theory.

Orlicz-Sobolev embeddings and applications to quasilinear elliptic equations in $\mathbb{R}^N$

Alessio Pomponio

In this talk we present an existence result for the quasilinear elliptic problem

$$\begin{cases} -\nabla \cdot \left[\varphi'(|\nabla u|^2)\nabla u\right] + |u|^{\alpha-2}u = |u|^{s-2}u, & x \in \mathbb{R}^N, \\ u(x) \to 0, \quad \text{as } |x| \to \infty, \end{cases}$$

where $\varphi(t)$ behaves like $t^{q/2}$ for small t and $t^{p/2}$ for large t, being $1 < p < q < N$, $1 < \alpha \leqslant p^* q'/p'$ and $\max\{q, \alpha\} < s < p^*$, $p^* =$

$\frac{pN}{N-p}$ and p' and q' are the conjugate exponents, respectively, of p and q.

Our aim is to approach the problem variationally by using the tools of critical points theory. One of the main difficulty consists in identifying the right functional setting for the problem. Indeed, the different growths at zero and at infinity of the principal part advise us not to use classical Sobolev spaces and to introduce a completely new functional framework. So, we have to define a sort of Orlicz-Sobolev space and we have to study the embedding properties of such space.

The results of this talk are contained in a recent joint work with A. Azzollini and P. d'Avenia.

Periodic solutions of a capillarity equation in the presence of lower and upper solutions

Sabrina Rivetti

We develop a lower and upper solutions method for the T-periodic problem associated with the capillarity equation

$$-\left(u' \,/\, \sqrt{1+u'^2}\right)' = f(t,u).$$

In the setting of bounded variation functions we prove the existence of T-periodic solutions both in the case where the lower solution α and the upper solution β satisfy $\alpha \leqslant \beta$, and in the case where $\alpha \nleqslant \beta$. In the former case we obtain, besides existence, also localization, regularity and order stability results. In the latter case we still prove existence of a solution, but now we need a control on f with respect to the first branch of the Dancer–Fučík spectrum of the T-periodic problem for the 1-Laplace operator.

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Asymptotic axial symmetry of solutions of parabolic equations in bounded radial domains

Alberto Saldaña

In this talk we present the notion of asymptotic (in time) symmetry for solutions of nonlinear parabolic boundary value problems. In this setting, a previous result will be reviewed and a new result is given: Under rather general assumptions, if the initial profile satisfy a reflection inequality with respect to a hyperplane containing the origin, then the solution is asymptotically foliated Schwarz symmetric, i.e., all elements in the associated omega limit set are axially symmetric with respect to a common axis passing through the origin and nonincreasing in the polar angle from this axis. A corollary for elliptic equations is also given. This a joint work with Tobias Weth.

A nonresonance condition for radial solutions of a nonlinear Neumann elliptic problem

Andrea Sfecci

I introduce an existence result for radial solutions of a Neumann elliptic problem whose nonlinearity asymptotically lies between the first two eigenvalues. The problem under consideration is

$$\begin{cases} -\Delta u = g(u) + e(|x|) & \text{in } B_1 \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial B_1 \,, \end{cases}$$

where $B_1 = \{x \in \mathbb{R}^N : |x| < 1\}$, $g : \mathbb{R} \to \mathbb{R}$ and $e : [0,1] \to \mathbb{R}$ are continuous functions, and $|\cdot|$ indicates the euclidean norm.

It was shown in [1] that the following condition is sufficient to guarantee the existence of a solution for the corresponding Dirichlet problem:

$$\liminf_{x\to-\infty} \frac{2G(s)}{s^2} < \frac{\pi^2}{4}\,, \quad \text{and} \quad \liminf_{x\to+\infty} \frac{2G(s)}{s^2} < \frac{\pi^2}{4}\,.$$

Notice that, denoting by λ_1 the first positive eigenvalue of $-\Delta$ with Dirichlet boundary conditions, one has that $\pi^2/4 < \lambda_1$, unless the dimension is equal to 1, in which case $B_1 = (-1,1)$ and $\lambda_1 = \pi^2/4$.

I will show how this assumption can be modified, preserving at least one of the "liminf" conditions, in order to obtain an existence result in the case of the Neumann problem, provided that a suitable condition at the first (zero) eigenvalue is also satisfied. It seems that such a result has not been carried out yet, not even in the case $N = 1$.

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Partial radial symmetry of positive solutions for semilinear elliptic equations in a disc and its application to the Hénon equation

Naoki Shioji

In this talk, we study a partial radial symmetry of positive solutions of

$$\begin{cases} \Delta u + f(|x|, u) = 0 & \text{in } D \setminus \{0\}, \\ u = 0 & \text{on } \partial D, \end{cases} \tag{1}$$

where $D = \{x = (x_1, x_2) \in \mathbb{R}^2 : |x| < 1\}$ and $f \in C((0, 1) \times (0, \infty), \mathbb{R})$ is a function such that $r \mapsto r^{2-2n} f(r, u) : (0, 1) \to \mathbb{R}$ is nonincreasing with $n \in \mathbb{N} \setminus \{1\}$, and we apply our result to the Hénon equation

$$\begin{cases} \Delta u + |x|^\alpha |u|^{p-2} u = 0 & \text{in } D, \\ u = 0 & \text{on } \partial D, \end{cases} \tag{2}$$

with $\alpha \in (0, \infty)$ and $p \in (2, \infty)$. We say a function $u : \overline{D} \to \mathbb{R}$ is n-mode ($n \in \mathbb{N}$) if

$$u(r \exp(i\theta)) = u\left(r \exp\left(i\left(\theta + \frac{2\pi}{n}\right)\right)\right)$$

for each $(r, \theta) \in [0, 1] \times \mathbb{R}$, where $r \exp(i\theta) = (r \cos\theta, r \sin\theta)$ for $r \geqslant 0$ and $\theta \in \mathbb{R}$. Now, we show our result.

Theorem. *Let $n \in \mathbb{N}$ with $n \geqslant 2$ and let $f \in C((0, 1) \times (0, \infty), \mathbb{R})$ such that*

1. *for each* $u \in (0,\infty)$, $r \mapsto r^{2-2n} f(r,u) : (0,1) \to \mathbb{R}$ *is nonincreasing,*

2. *for each* $r_0 \in (0,1)$ *and* $M \in (0,\infty)$,

$$\sup\left\{ \left| \frac{f(r,u_1) - f(r,u_2)}{u_1 - u_2} \right| : \right.$$
$$\left. (r,u_1,u_2) \in (r_0,1) \times (0,M]^2, u_1 \neq u_2 \right\} < \infty.$$

Let $u \in C^2(D \setminus \{0\}) \cap C(\overline{D})$ *be an* n*-mode positive solution of* (1) *such that* u *is of class* C^n *at the origin. Then* u *is radially symmetric and* $u_r(|x|) < 0$ *for* $r = |x| \in (0,1)$.

For each $\alpha > 0$, we denote by $\lceil \alpha \rceil$ and $\lfloor \alpha \rfloor$ the smallest integer greater than or equal to α and the largest integer less than or equal to α, respectively. For each $\alpha \geqslant 0$ and $p > 2$, we set

$$R_{\alpha,p}(u) = \frac{\int_D |\nabla u|^2 \, dx}{\left(\int_D |x|^\alpha |u|^p \, dx\right)^{2/p}} \quad \text{for each } u \in H_0^1(D) \setminus \{0\}.$$

THEOREM. *There hold the following.*

1. *If* $\alpha \in (0,\infty)$, $p \in (2,\infty)$ *and* u *is an* m*-mode, positive solution of* (2) *with* $1 + \lceil \alpha/2 \rceil \leqslant m \leqslant k_\alpha$, *then* u *is radially symmetric, where*

$$k_\alpha = \begin{cases} \lfloor \alpha \rfloor + 2 & \text{if } \alpha \in (0,\infty) \setminus \mathbb{N}, \\ \alpha + 1 & \text{if } \alpha \text{ is an odd natural number,} \\ \infty & \text{if } \alpha \text{ is an even natural number.} \end{cases}$$

2. *For each* $\alpha \in (2,\infty)$, *if* $p \in (2,\infty)$ *is large enough, then problem* (2) *has a nonradial,* l*-mode positive solution* u_l *for* $l = 1, \ldots, \lceil \alpha/2 \rceil$ *satisfying* $R_{\alpha,p}(u_1) < \cdots < R_{\alpha,p}(u_l)$.

Remark. For each $p \in (2,\infty)$ and $m \in \mathbb{N}$, the number of nonradial position solutions of (2) tends to infinity as $\alpha \to \infty$.

This is a joint work with Professor Kohtaro Watanabe.

On damped semilinear wave equations with singularly perturbed boundary conditions

Joseph Shomberg

Under consideration is the damped semilinear wave equation

$$u_{tt} + u_t - \Delta u + u + f(u) = 0$$

on a bounded domain Ω in $\mathbb{R}^3$ with a perturbation parameter $\varepsilon > 0$ occurring in an acoustic boundary condition, limiting ($\varepsilon = 0$) to a Robin boundary condition. With minimal assumptions on the nonlinear term f, the existence and uniqueness of global weak solutions is shown. Also, the existence of a family of global attractors is shown to exist. After proving a general result concerning the robustness of a one-parameter family of sets, the result is applied to the family of global attractors. Because of the complicated boundary conditions for the perturbed problem, fractional powers of the Laplacian are not well-defined; moreover, because of the restrictive growth assumptions on f, the family of global attractors is obtained from the asymptotic compactness method developed by J. Ball for generalized semiflows.

Schrödinger-Poisson system with non constant interaction [9]

Gaetano Siciliano

We consider the following system of Schrödinger-Poisson type in a bounded and regular domain $\Omega \subseteq \mathbb{R}^3$:

$$\begin{cases} -\Delta u + q\varphi u = \omega u & \text{in } \Omega, \\ -\Delta\varphi = qu^2 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \\ \frac{\partial\varphi}{\partial\mathbf{n}} = h & \text{on } \partial\Omega, \end{cases}$$

where q is a given continuous function defined in $\overline{\Omega}$ (representing the charge density), h is a given (and smooth) function on

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$\partial\Omega$ and $\omega \in \mathbb{R}$. This system arises from the search of stationary solutions $\psi(x,t) = u(x)e^{i\omega t}$ for the Schrödinger equation coupled with the Maxwell equations in the purely electrostatic case. We are assuming that u and ω are both unknowns. The other unknown $\varphi(x)$ is interpreted as the electrostatic potential generated by the motion of the particle in the region Ω. Actually we are interested in solutions with u normalized in L^2, i.e.

$$\int_\Omega u^2 \, \mathrm{d}x = 1.$$

Note that another constraint appears naturally since the Neumann condition implies that the following necessary condition has to be satisfied by any solution:

$$\int_{\partial\Omega} h \, \mathrm{d}\sigma = \int_\Omega q \, u^2 \, \mathrm{d}x.$$

This kind of problem has been introduced by Benci and Fortunato in [1]. However they consider a Dirichelet boundary condition also on φ while q is taken as a constant. The problem with the Neumann boundary condition on φ has been studied in [2] but again, as in [1], under the hypothesis $q = constant$. This actually has motivated our investigation in the case $q = q(x) \neq constant$: we are able to prove that under a suitable condition on the boundary datum h, the Ljusternick-Schnirelmann theory can be employed and the problem admits infinitely many solutions (see [3]).

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Boundary value problems for quasi-linear elliptic second order equations in unbounded cone-like domains

Damian Wiśniewski

Let $B_1(\mathcal{O})$ be the unit ball in $\mathbb{R}^n$, $n \geqslant 2$ with center at the origin $\mathcal{O}$ and $G \subseteq \mathbb{R}^n \setminus B_1(\mathcal{O})$ be an unbounded domain with the smooth boundary ∂G. We assume that $G = G_0 \cup G_R$, where G_0 is a bounded domain in $\mathbb{R}^n$ and $G_R = \{(r, \omega) \in \mathbb{R}^n | \ r \in (R, \infty), \ \omega \in \Omega \subseteq S^{n-1}, \ n \geqslant 2\}$, where $R \gg 1$ and S^{n-1} is the unit sphere.

We investigate the behaviour of weak solutions to the boundary value problems for elliptic divergence quasi-linear equations in a neighborhood of infinity:

$$\begin{cases} -\dfrac{\mathrm{d}}{\mathrm{d}x_i} a_i(x, u, \nabla u) + b(x, u, \nabla u) = 0, & x \in G; \\ \alpha(x)\dfrac{\partial u}{\partial \nu} + \dfrac{\gamma(\omega)}{|x|^{m-1}} u \cdot |u|^{q+m-2} = g(x, u), & x \in \partial G; \\ \lim\limits_{|x| \to \infty} u(x) = 0; & \end{cases}$$

here: $a_i : G \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$, $b : G \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$; $m > 1$, $q \geqslant 0$, $\alpha(x) = \begin{cases} 0, & if \ x \in \mathcal{D}; \\ 1, & if \ x \notin \mathcal{D}, \end{cases}$ $\mathcal{D} \subseteq \partial G$ is the part of the boundary ∂G, where the Dirichlet boundary condition is posed; $\frac{\partial u}{\partial \nu} = a_i(x, u, \nabla u) \cos(\vec{n}, x_i)$, $\vec{n}$ denotes the unit outward with respect to G normal to ∂G.

We establish estimates of weak solutions of the type $u(x) = O(r^{\alpha})$.

We use the Kondratiev ring method and the method of integro-differential inequalities.

On a ‘balance’ condition for a class of PDEs including porous medium and chemotaxis effect [10]

Anna Zhigun

In this talk we consider a class of degenerate parabolic systems that encompasses two different effects: porous medium and chemotaxis. Such classes of equations arise in the mesoscale level modelling of biomass spreading mechanisms via chemotaxis. We address the well-posedness, the uniform boundedness in time and space and the long time dynamics under certain 'balance' condition on the order of the porous medium degeneracy and the growth of the chemotactic function.

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4 Sponsors

5 Practical information

5.1 Lunches

For lunch, there are several options. You can have a sandwich or chose between several *plats du jour* at the cafétéria Sodexo in front of the auditorium. There you will find every day an option for vegetarian diet. The cafétéria is closed on Saturday.

There are other options at walking distance from the campus. Several cafés or restaurants offer lunchtime specials. The more attractive ones are located in the *Avenue de l'Université* or in the *Chaussée de Boondael*, starting from the entrance of the local cemetery. These streets are highlighted in black on the map below.

5.2 Dinners

For dinners, you will find plenty of restaurants around the *Grand Place*. Brussels offers an international and eclectic choice of restaurants. More typical but rather touristic restaurants can be found in the *Rue des bouchers* and *Petite rue des bouchers* close to the Grand Place. Good alternatives are the *Quartier St Géry* and the *Place St Catherine* at walking distance from the *Grand Place*. Restaurants at the *Place St Catherine* are specialized in seafood and fish. Prices may vary a lot.

For quicker and more economical deals, you may try the kebabs, pasta or pizza in the *Rue Marché aux fromages* close to the *Grand Place*.

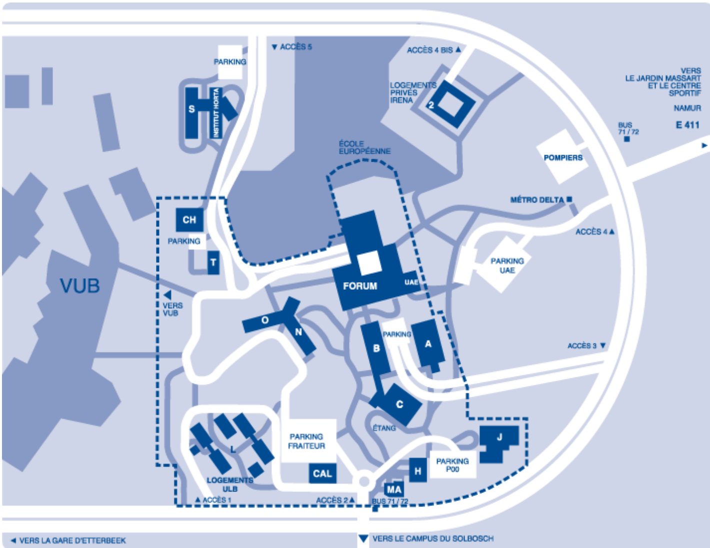
5.3 WiFi

The network is called "Plaine-WiFi". Login and password will be available from the organizers during the meeting. You have to start your session in a browser. Good signal quality is available in the rooms NO4.007 and NO4.006 (4th floor of building NO).

You may also be able to use "URBIZONE" which is a free network sponsored by the city of Brussels. After associating with the access point, you need to launch a browser and follow the instructions to create an account.

6 Maps

Map of the campus:



Streets where to have lunch: